Dear students; I wish you best of luck for your final TU exam. I have prepared these 5 sets to prepare Mathematics I. You solve these sets as an "Final Assignment" as a part of your practical marks

St. Xavier's College

Bachelor of Science in Computer Science and Information Technology

Semester: First **Course No:MTH-112**

Course Title: Mathematics I

Send up examination 2018(2074)

Sub: Mathematics Full Marks: 80
Time: 3 hrs Pass Marks: 32
Candidates are required to give their answers in their own words as far as practicable.

Set1

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x) = \begin{cases} 1-x, & if \ \ x \le -1 \\ x^2 & if \ \ x > -1 \end{cases}$ [5]

Evaluate f(-3); f(-1) and f(0) and sketch the graph.

(b) Prove that the limit $\lim\limits_{x \to 0} \frac{|x|}{x}$ does not exist. [5]

2. (a) Sketch the curve : $\frac{2x^2}{x^2-1}$ [5]

(b) Estimate the area between the curve $y = e^x$ and the lines x = 0 and x = 1, using rectangle method. [5]

3. (a) Show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. [4]

(b) Define initial value problem. Solve: $x^2 y'' + xy' = 1,\ x > 0\ ,\ y'(1) = 0\ and\ \ y(1) = 0.$ [6]

4. (a) Find the torsion, normal and curvature of helix curve: $(2\cos t)\vec{i} + (2\sin t)\vec{j} + 3t\,\vec{k}$. [6]

(b) Show that the curvature of a circle of radius a is 1/a. [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $y = x^5 + x$ (b) $y = 1 - x^4$ (c) $y = 2x - x^2$

6. Define continuity of a function at a point x = a. Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuous on the interval [-1, 1].

7. Verify the Rolle's theorem for $f(x) = x^3 - x^2 - 6x + 2$ in [0, 3].

8. Find the third approximation x_3 to the root of the equation $f(x) = x^3 - 6x - 5$ setting $x_0 = 2$.

9. Evaluate: $\int_{-\infty}^{0} x\, e^{x} dx.$

10. Find the volume of the solid obtained by about y - axis the region between y = x and y = x^2.

11. Solve: $y''+y'-6y=0,\ x>0\ ,\ y'(0)=0\ and\ \ y(0)=1.$

12. Find the curvature of the twisted cubic $\vec{r}(t)=\langle t, t^2, t^3 \rangle$ at a general point and at (0, 0, 0)

13. Show that the series $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ converges.

14. Calculate: $\iint_R f(x,y)\,dA$ for r = $f(x,y)=x^2y-2xy,\quad R:-2\le x\le 0,\ \ 0\le y\le 3.$

15. Find the partial derivatives of $f(x,y)=x^3+x^2y^3-2y^2$ at (2, 1).

Set2

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x)=\begin{cases} 1+x, & if\ \ x\le 1 \\ x^2 & if\ \ x>1 \end{cases}$ [5]

Evaluate f(3); f(1) and f(0) and sketch the graph.

(b) Prove that the limit $x \xrightarrow{\lim} 0\ |x|$ exists then find its value [5]

2. (a) Sketch the curve : $f(x)=\frac{x^2}{\sqrt{x+1}}.$ [5]

(b) Estimate the area between the curve $y=x^2$ and the lines x = 0 and x = 1, using rectangle method. [5]

3. (a) Find the volume of the solid obtained by rotating about the *x*-axis the region under the curve $y=\sqrt{x}$ from 0 to 1. [4]

(b) Define order of a differential equation.

Solve: $\frac{du}{dt}=\frac{2t+\sec^2 t}{2u}\quad , u(0)=-5.$ [6]

4. (a) Find the the unit normal and binormal vectors for the circular helix $(\cos t)\vec{i}\ +(\sin t)\ \vec{j}\ +t\vec{k}$. [6]

(b) Show that $x^2+y^2+z^2+4x-6y+2z+6=0$ is the equation of a sphere, and find its center and radius. [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $y=x^5+x$ (b) $y=1-x^4$ (c) $y=2x-x^2$

6. Find an equation of the tangent line to the parabola $y=2x-x^2$ at the point P(1, 1).

7. Where is the function $f(x)=|x|$ differentiable?

8. Starting with $x_1=2$, find the third approximation x_3 to the root of the equation $f(x)=x^3-2x-5$.

9. State Net change theorem. A particle moves along a line so that its velocity at time t is $v(t)=t^2-t-6$ (measured in meters per second).

(a) Find the displacement of the particle during the time period $1 \le t \le 4$.

(b) Find the distance traveled during this time period.

10. Find the length of the arc of the semi-cubical parabola $y^2 = x^3$ between the points (1, 1) and (4, 8) .

11. Define homogeneous second-order linear differential equation.
    Solve: $y'' + y = 0,\ x > 0\ ,\ y'(0) = 3\ and\ \ y(0) = 2.$

12. Find a vector perpendicular to the plane that passes through the points P(1, 4, 6), Q(-2, 5, -1) and R(1, -1, 1).

13. Determine whether the series $\sum_{n=1}^{\infty} \frac{5}{2n^2 + 4n + 3}$ converges or diverges.

14. Evaluate the iterated integrals. $\int_0^3 \int_1^2 x^2 y\,dy\,dx$ and $\int_1^2 \int_0^3 x^2 y\,dx\,dy.$

15. Find the length of the arc of the circular helix with vector equation
    $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + t\vec{k}$ from the point (1, 0, 0) to the point (1, 0, 2Π).

Set3

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x) = \begin{cases} x + 2, & if\ \ x < 0 \\ 1 - x & if\ \ x \ge 0 \end{cases}$ [5]

   Find domain and sketch the graph.

   (b) Evaluate the limit $\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$. [5]

2. (a) Sketch the curve : $f(x) = xe^x$. [5]

   (b) Estimate the area of the region bounded above by $y = e^x$, bounded below by y = x and bounded on the sides by x = 0 and x = 1. [5]

3. (a) Find the volume of the solid obtained by rotating the region bounded by , $y = x^3$, y = 8 and x = 0 about the y-axis. [4]

   (b) Define degree of a differential equation. Solve the differential equation : $\frac{dy}{dx} = \frac{x^2}{y^2}$ and hence find the solution of this equation that satisfies the initial condition y(0) = 2. [6]

4. (a) Find a vector equation and parametric equations for the line that passes through the point (5, 1, 3) and is parallel to the vector $\vec{i} + 4\vec{j} - 2\vec{k}$. Find two other points on the line. [6]

   (b) If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 7\vec{k}$, find the unit vector along $2\vec{a} + 3\vec{b}$. [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.
   (a) $y = x^5 + x$ (b) $y = 1 - x^4$ (c) $y = 2x - x^2$

6. Find the horizontal and vertical asymptotes of the graph of the function $f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$.

7. Suppose that f(0) = -3 and $f'(x) \le 5$ for all values of x . How large can f(2) possibly be?

8. Find the curvature of the parabola $y = x^2$ at the points (0, 0), (1, 1) and (2, 4)

9. Evaluate: $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx.$

10. Find the length of the arc of the parabola $y^2 = x$ from the points (0, 0) and (1, 1) .

11. Define non-homogeneous second-order linear differential equation.

Solve: $y''+y'-2y = x^2$.

12. Find the Maclaurin series of the function $f(x) = e^x$ and its radius of convergence.

13. Find the distance between two parallel planes 10x + 2y - 2z = 5 and 5x + y - z = 1.

14. Evaluate: $\iint_R f(x, y)\,dA$ for $f(x, y) = y\sin(xy)$, $R = [1, 2] \times [0, \pi]$.

15. Use the scalar triple product to show that the vectors, $\vec{a} = \vec{i} + 4\vec{j} - 7\vec{k}$, $\vec{a} = 2\vec{i} - \vec{j} + 4\vec{k}$ and $\vec{a} = -9\vec{j} + 18\vec{k}$ are coplanar.

Set4

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x) = \dfrac{3x + |x|}{x}$ [5]

Evaluate domain and sketch the graph.

(b) If $f(x) = \begin{cases} \sqrt{x-4}, & if \;\; x > 4 \\ 8 - 2x & if \;\; x < 4 \end{cases}$, determine whether limit exits or not. [5]

2. (a) Sketch the curve : $f(x) = \dfrac{\cos x}{2 + \sin x}$. [5]

(b) Estimate the area between the curve $y = \dfrac{1}{x}$ and the lines x = 0 and x = 1,

using Trapezoidal rule. [5]

3. (a) The region enclosed by the curves y = x and $y = x^2$ and is rotated about the x-axis. Find the volume of the resulting solid. [4]

(b) Define orthogonal trajectories. Find the orthogonal trajectories of the family of curves $x = ky^2$,where k is an arbitrary constant. [6]

4. (a) Find parametric equations and symmetric equations of the line that passes through the points A(2, 4, -3) and B(3, -1, 1). At what point does this line intersect the -plane? [6]

(b) Find the direction angles of the vector < 1, 2, 3> . [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $y = x^5 + x$ (b) $y = 1 - x^4$ (c) $y = 2x - x^2$

6. Define continuity of a function at a point x = a. Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuous on the interval [-1, 1].

7. Evaluate the following, using L Hospital's Rule $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$.

8. A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

9. Evaluate: $\int_0^1 \ln x \, dx$.

10. Find the arc length function for the curve $y = x^2 - \frac{1}{8}\ln x$ taking the point P(1, 1) as the starting point.

11. What is the solution of non-homogeneous second-order linear differential equation. Solve: $y'' - 4y = xe^x + \cos 2x$.

12. Find the Maclaurin's series for the function f(x) = x.cosx.

13. Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right)$.

14. Find the angle between the planes x + y + z = 1 and x - 3y + 2z = 1.Find symmetric equations for the line of intersection L of these two planes.

15. Find the second partial derivatives of $f(x, y) = x^3 + x^2y^3 - 2y^2$.

Set5

Group A **(3 x 10 = 30)**

Attempt any three questions.

1. (a) A function is defined by $f(x) = \sqrt{x-5}$ [5]
Find domain and sketch the graph.

(b) Prove that the limit $\lim_{x \to 0} x^2 \sin\frac{1}{x} = 0$. [5]

2. (a) Sketch the curve : $f(x) = \ln(4 - x^2)$. [5]

(b) Estimate the area between the curve $y = e^{x^2}$ and the lines x = 0 and x = 1, using Simpson's rule. [5]

3. (a) Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and y = 0. [4]

(b) Define differential equation. Solve: $x \ln x = y(1 + \sqrt{3 + y^2})\, y'$ if y(1) = 1. [6]

4. (a) Show that the lines and with parametric equations

x = 1 + t   y = - 2 + 3t   z = 4 – t

x = 2s   y = 3 + s   z = - 4 + 4s   are **skew lines**; that is, they do not intersect and are not parallel (and therefore do not lie in the same plane) [6]

(b) Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2\rangle$ onto $\vec{a} = \langle -2, 3, 1\rangle$ [4]

Group B **(10 x 5 = 50)**

Attempt any ten questions.

5. Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $y = x^5 + x$ (b) $y = 1 - x^4$ (c) $y = 2x - x^2$

6. Define indeterminate forms.Evaluate the following: $\lim_{x \to 0^+} x^x$.

7. Find an equation of the plane that passes through the points P(1, 3, 2), Q(3, -1, 6) and R(5, 2, 0).

8. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a

straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

9. For what value of P, the integral $\int_1^\infty \frac{1}{x^P}dx$ is convergent?

10. The curve $y=\sqrt{4-x^2}$, $-1\le x\le 1$, is an arc of the circle $x^2+y^2=4$. Find the area of the surface obtained by rotating this arc about the x-axis.

11. Solve: $y''+y=e^x+x^3, \quad y'(0)=0 \text{ and } y(0)=2.$

12. Find the point at which the line with parametric equations,

$x = 2 + 3t \qquad y = -4t \qquad z = 5 + t$, intersects the plane $4x + 5y - 2z = 18$.

13. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(1+n)}$ converges, find its sum.

14. Find the volume of the solid that is bounded by the elliptic paraboloid $x^2+2y^2+z=16$, the planes x = 2 and y = 2 , and the three coordinate planes.

15. Find an equation of the plane through the point (2, 4, -1) with normal vector $\langle 2,3,4\rangle$. Find the intercepts and sketch the plane.

Best of luck!!!!!!